Multi-headed Lattice Green Function (N = 5, M = 2)

REC for $r_{2,5}(n)$ in Theorem 6.2

Out[ ]=

$$\begin{aligned}
&\big(42\,140\,738\,676\,326\,400\,000 + 157\,842\,901\,249\,818\,624\,000\,\alpha + 257\,331\,505\,709\,737\,574\,400\,\alpha^{2} + \\
&\quad 243\,764\,108\,399\,982\,673\,920\,\alpha^{3} + 150\,397\,023\,447\,243\,816\,960\,\alpha^{4} + 63\,968\,341\,924\,254\,842\,880\,\alpha^{5} + \\
&\quad 19\,301\,263\,998\,729\,584\,640\,\alpha^{6} + 4\,174\,508\,253\,346\,529\,280\,\alpha^{7} + 643\,779\,101\,841\,162\,240\,\alpha^{8} + \\
&\quad 69\,168\,932\,868\,587\,520\,\alpha^{9} + 4\,922\,454\,740\,828\,160\,\alpha^{10} + 208\,614\,614\,630\,400\,\alpha^{11} + 3\,986\,266\,521\,600\,\alpha^{12}\big) \\
&\ \mathrm{Seq}[\alpha] + \big(118\,427\,858\,324\,029\,440\,000 + 355\,246\,559\,316\,108\,902\,400\,\alpha + \\
&\quad 481\,552\,669\,599\,250\,186\,240\,\alpha^{2} + 390\,301\,079\,007\,991\,857\,152\,\alpha^{3} + \\
&\quad 210\,764\,527\,991\,633\,575\,936\,\alpha^{4} + 79\,918\,506\,618\,774\,847\,488\,\alpha^{5} + 21\,826\,970\,852\,964\,532\,224\,\alpha^{6} + \\
&\quad 4\,327\,696\,049\,218\,387\,968\,\alpha^{7} + 618\,429\,092\,691\,574\,784\,\alpha^{8} + 62\,134\,020\,238\,999\,552\,\alpha^{9} + \\
&\quad 4\,167\,373\,533\,741\,056\,\alpha^{10} + 167\,578\,215\,383\,040\,\alpha^{11} + 3\,056\,137\,666\,560\,\alpha^{12}\big)\ \mathrm{Seq}[1+\alpha] + \\
&\big(62\,676\,619\,662\,919\,680\,000 + 168\,213\,967\,990\,385\,049\,600\,\alpha + 205\,820\,392\,167\,964\,974\,080\,\alpha^{2} + \\
&\quad 151\,791\,584\,110\,964\,534\,272\,\alpha^{3} + 75\,137\,340\,688\,642\,841\,600\,\alpha^{4} + 26\,295\,232\,911\,598\,126\,080\,\alpha^{5} + \\
&\quad 6\,670\,149\,766\,003\,083\,264\,\alpha^{6} + 1\,235\,525\,904\,487\,723\,008\,\alpha^{7} + 165\,841\,646\,014\,996\,480\,\alpha^{8} + \\
&\quad 15\,729\,900\,132\,270\,080\,\alpha^{9} + 1\,000\,638\,108\,860\,416\,\alpha^{10} + 38\,329\,059\,901\,440\,\alpha^{11} + 668\,530\,114\,560\,\alpha^{12}\big) \\
&\ \mathrm{Seq}[2+\alpha] + \big(1\,794\,185\,247\,360\,768\,000 + 4\,260\,839\,636\,091\,043\,840\,\alpha + 4\,649\,746\,903\,477\,813\,888\,\alpha^{2} + \\
&\quad 3\,082\,953\,754\,682\,083\,328\,\alpha^{3} + 1\,382\,952\,049\,413\,254\,272\,\alpha^{4} + 442\,032\,317\,052\,873\,728\,\alpha^{5} + \\
&\quad 103\,190\,706\,316\,889\,344\,\alpha^{6} + 17\,720\,524\,544\,509\,952\,\alpha^{7} + 2\,220\,812\,336\,954\,368\,\alpha^{8} + \\
&\quad 198\,014\,286\,036\,992\,\alpha^{9} + 11\,919\,389\,769\,728\,\alpha^{10} + 434\,786\,795\,520\,\alpha^{11} + 7\,266\,631\,680\,\alpha^{12}\big)\ \mathrm{Seq}[3+\alpha] + \\
&\big(-3\,522\,851\,180\,688\,416\,000 - 8\,446\,568\,365\,407\,735\,680\,\alpha - 9\,248\,095\,565\,260\,356\,576\,\alpha^{2} - \\
&\quad 6\,114\,775\,140\,268\,882\,576\,\alpha^{3} - 2\,719\,484\,985\,845\,017\,792\,\alpha^{4} - 857\,108\,315\,069\,629\,104\,\alpha^{5} - \\
&\quad 196\,310\,820\,429\,867\,616\,\alpha^{6} - 32\,924\,151\,546\,376\,000\,\alpha^{7} - 4\,013\,146\,001\,886\,336\,\alpha^{8} - \\
&\quad 346\,719\,870\,364\,160\,\alpha^{9} - 20\,154\,401\,039\,360\,\alpha^{10} - 707\,739\,648\,000\,\alpha^{11} - 11\,354\,112\,000\,\alpha^{12}\big)\ \mathrm{Seq}[4+\alpha] + \\
&\big(-458\,904\,717\,778\,020\,000 - 1\,056\,134\,626\,035\,848\,800\,\alpha - 1\,109\,896\,707\,061\,337\,856\,\alpha^{2} - \\
&\quad 704\,344\,314\,090\,018\,780\,\alpha^{3} - 300\,647\,030\,233\,781\,612\,\alpha^{4} - 90\,944\,593\,157\,694\,708\,\alpha^{5} - \\
&\quad 19\,993\,089\,019\,041\,540\,\alpha^{6} - 3\,218\,776\,240\,146\,608\,\alpha^{7} - 376\,681\,142\,235\,984\,\alpha^{8} - \\
&\quad 31\,252\,297\,558\,272\,\alpha^{9} - 1\,745\,103\,671\,296\,\alpha^{10} - 58\,889\,994\,240\,\alpha^{11} - 908\,328\,960\,\alpha^{12}\big)\ \mathrm{Seq}[5+\alpha] + \\
&\big(-1\,106\,658\,753\,555\,600 - 2\,330\,306\,062\,592\,328\,\alpha - 2\,249\,741\,897\,564\,436\,\alpha^{2} - 1\,317\,143\,965\,540\,014\,\alpha^{3} - \\
&\quad 520\,970\,340\,108\,810\,\alpha^{4} - 146\,691\,130\,015\,168\,\alpha^{5} - 30\,156\,685\,922\,334\,\alpha^{6} - 4\,561\,556\,620\,082\,\alpha^{7} - \\
&\quad 503\,951\,197\,636\,\alpha^{8} - 39\,663\,617\,640\,\alpha^{9} - 2\,111\,344\,496\,\alpha^{10} - 68\,259\,840\,\alpha^{11} - 1\,013\,760\,\alpha^{12}\big)\ \mathrm{Seq}[6+\alpha] + \\
&\big(836\,209\,651\,013\,100 + 1\,823\,470\,291\,632\,528\,\alpha + 1\,811\,702\,917\,816\,029\,\alpha^{2} + 1\,084\,613\,257\,235\,718\,\alpha^{3} + \\
&\quad 435\,833\,439\,807\,171\,\alpha^{4} + 123\,860\,858\,052\,324\,\alpha^{5} + 25\,531\,982\,914\,119\,\alpha^{6} + 3\,847\,089\,898\,422\,\alpha^{7} + \\
&\quad 420\,608\,699\,769\,\alpha^{8} + 32\,547\,074\,928\,\alpha^{9} + 1\,692\,297\,492\,\alpha^{10} + 53\,095\,680\,\alpha^{11} + 760\,320\,\alpha^{12}\big)\ \mathrm{Seq}[7+\alpha]
\end{aligned}$$