Multi-headed Lattice Green Function (N = 4, M = 3)

REC for $\tilde{r}_{3,4}(n)$ in Theorem 4.3

Out[∘]=

$$
\begin{aligned}
&\big(221\,086\,792\,032\,258\,663\,383\,040 + 3\,002\,581\,182\,281\,579\,476\,549\,632\,\alpha +\\
&\quad 18\,896\,284\,453\,973\,181\,469\,818\,880\,\alpha^{2} + 73\,337\,056\,136\,834\,742\,984\,114\,176\,\alpha^{3} +\\
&\quad 197\,017\,275\,538\,043\,925\,583\,364\,096\,\alpha^{4} + 389\,745\,626\,428\,476\,129\,286\,291\,456\,\alpha^{5} +\\
&\quad 589\,529\,476\,016\,351\,811\,509\,157\,888\,\alpha^{6} + 698\,690\,177\,713\,813\,455\,561\,031\,680\,\alpha^{7} +\\
&\quad 659\,396\,154\,092\,196\,671\,988\,432\,896\,\alpha^{8} + 500\,766\,687\,956\,261\,350\,615\,810\,048\,\alpha^{9} +\\
&\quad 307\,887\,490\,552\,535\,839\,569\,608\,704\,\alpha^{10} + 153\,616\,793\,330\,862\,792\,246\,296\,576\,\alpha^{11} +\\
&\quad 62\,125\,104\,506\,185\,984\,379\,977\,728\,\alpha^{12} + 20\,265\,270\,278\,609\,884\,774\,662\,144\,\alpha^{13} +\\
&\quad 5\,282\,843\,409\,745\,454\,510\,899\,200\,\alpha^{14} + 1\,084\,193\,901\,809\,507\,676\,192\,768\,\alpha^{15} +\\
&\quad 171\,154\,981\,038\,855\,165\,050\,880\,\alpha^{16} + 20\,040\,031\,539\,432\,857\,272\,320\,\alpha^{17} +\\
&\quad 1\,638\,003\,152\,561\,664\,688\,128\,\alpha^{18} + 83\,373\,097\,696\,100\,352\,000\,\alpha^{19} + 1\,988\,330\,027\,074\,191\,360\,\alpha^{20}\big)\\
&\ \mathrm{Seq}[\alpha] + \big(-123\,596\,648\,884\,357\,621\,088\,256 - 1\,387\,410\,081\,329\,207\,115\,251\,712\,\alpha -\\
&\quad 7\,308\,010\,505\,383\,031\,273\,947\,136\,\alpha^{2} - 24\,020\,604\,752\,075\,269\,740\,691\,456\,\alpha^{3} -\\
&\quad 55\,262\,591\,055\,735\,725\,773\,815\,808\,\alpha^{4} - 94\,607\,549\,345\,038\,165\,436\,006\,400\,\alpha^{5} -\\
&\quad 125\,070\,786\,847\,359\,746\,869\,821\,440\,\alpha^{6} - 130\,760\,992\,638\,503\,780\,446\,109\,696\,\alpha^{7} -\\
&\quad 109\,819\,712\,522\,499\,293\,630\,693\,376\,\alpha^{8} - 74\,830\,049\,897\,678\,615\,099\,736\,064\,\alpha^{9} -\\
&\quad 41\,599\,115\,200\,046\,517\,939\,601\,408\,\alpha^{10} - 18\,902\,277\,196\,351\,684\,209\,803\,264\,\alpha^{11} -\\
&\quad 7\,008\,965\,526\,989\,775\,347\,122\,176\,\alpha^{12} - 2\,109\,519\,207\,312\,665\,281\,560\,576\,\alpha^{13} -\\
&\quad 510\,375\,764\,108\,304\,797\,663\,232\,\alpha^{14} - 97\,744\,104\,267\,386\,959\,429\,632\,\alpha^{15} -\\
&\quad 14\,472\,279\,363\,085\,494\,386\,688\,\alpha^{16} - 1\,596\,811\,738\,769\,963\,089\,920\,\alpha^{17} - 123\,530\,156\,260\,699\,668\,480\\
&\quad \alpha^{18} - 5\,975\,058\,303\,292\,538\,880\,\alpha^{19} - 135\,920\,997\,944\,524\,800\,\alpha^{20}\big)\ \mathrm{Seq}[1+\alpha] +\\
&\big(2\,413\,729\,498\,666\,800\,513\,024 + 25\,435\,086\,835\,865\,925\,058\,560\,\alpha + 125\,542\,481\,225\,411\,227\,975\,680\,\alpha^{2} +\\
&\quad 386\,097\,946\,352\,750\,392\,590\,336\,\alpha^{3} + 830\,183\,396\,028\,360\,968\,208\,384\,\alpha^{4} +\\
&\quad 1\,327\,255\,653\,860\,270\,011\,465\,728\,\alpha^{5} + 1\,637\,850\,112\,836\,596\,110\,688\,256\,\alpha^{6} +\\
&\quad 1\,598\,197\,760\,043\,557\,807\,628\,288\,\alpha^{7} + 1\,252\,980\,911\,862\,994\,173\,739\,008\,\alpha^{8} +\\
&\quad 797\,358\,770\,338\,813\,407\,952\,896\,\alpha^{9} + 414\,276\,959\,391\,975\,941\,603\,328\,\alpha^{10} +\\
&\quad 176\,103\,421\,096\,866\,815\,410\,176\,\alpha^{11} + 61\,159\,515\,859\,482\,838\,548\,480\,\alpha^{12} +\\
&\quad 17\,263\,930\,413\,062\,410\,149\,888\,\alpha^{13} + 3\,923\,295\,133\,237\,310\,914\,560\,\alpha^{14} +\\
&\quad 706\,924\,713\,366\,338\,125\,824\,\alpha^{15} + 98\,652\,029\,401\,005\,981\,696\,\alpha^{16} + 10\,278\,087\,291\,823\,325\,184\,\alpha^{17} +\\
&\quad 752\,234\,327\,699\,226\,624\,\alpha^{18} + 34\,490\,272\,274\,841\,600\,\alpha^{19} + 745\,214\,176\,788\,480\,\alpha^{20}\big)\ \mathrm{Seq}[2+\alpha] +\\
&\big(-9\,569\,617\,440\,812\,835\,840 - 97\,443\,791\,378\,162\,009\,856\,\alpha - 463\,583\,339\,186\,644\,316\,800\,\alpha^{2} -\\
&\quad 1\,370\,837\,922\,368\,778\,354\,176\,\alpha^{3} - 2\,827\,452\,328\,200\,593\,850\,560\,\alpha^{4} - 4\,326\,575\,055\,112\,730\,856\,640\,\alpha^{5} -\\
&\quad 5\,099\,519\,612\,920\,329\,528\,000\,\alpha^{6} - 4\,743\,666\,552\,937\,883\,189\,952\,\alpha^{7} - 3\,539\,068\,890\,050\,114\,722\,112\,\alpha^{8} -\\
&\quad 2\,139\,750\,587\,880\,300\,657\,856\,\alpha^{9} - 1\,054\,730\,779\,373\,468\,537\,920\,\alpha^{10} - 424\,824\,967\,934\,147\,228\,480\,\alpha^{11} -\\
&\quad 139\,643\,546\,214\,642\,867\,648\,\alpha^{12} - 37\,274\,084\,807\,088\,072\,384\,\alpha^{13} - 8\,003\,802\,897\,605\,020\,608\,\alpha^{14} -\\
&\quad 1\,361\,866\,764\,260\,304\,576\,\alpha^{15} - 179\,386\,646\,751\,384\,192\,\alpha^{16} - 17\,635\,678\,788\,631\,680\,\alpha^{17} -\\
&\quad 1\,217\,772\,669\,657\,600\,\alpha^{18} - 52\,679\,537\,809\,920\,\alpha^{19} - 1\,074\,030\,451\,200\,\alpha^{20}\big)\ \mathrm{Seq}[3+\alpha] +\\
&\big(9\,051\,531\,325\,562\,880 + 90\,332\,029\,095\,081\,984\,\alpha + 420\,333\,410\,362\,428\,416\,\alpha^{2} +\\
&\quad 1\,213\,206\,945\,955\,473\,664\,\alpha^{3} + 2\,437\,377\,188\,874\,087\,136\,\alpha^{4} + 3\,625\,291\,113\,645\,770\,712\,\alpha^{5} +\\
&\quad 4\,144\,688\,219\,837\,114\,384\,\alpha^{6} + 3\,731\,957\,019\,300\,871\,994\,\alpha^{7} + 2\,689\,507\,840\,271\,682\,912\,\alpha^{8} +\\
&\quad 1\,567\,534\,832\,320\,365\,967\,\alpha^{9} + 743\,334\,125\,295\,350\,476\,\alpha^{10} + 287\,455\,002\,784\,035\,524\,\alpha^{11} +\\
&\quad 90\,539\,774\,552\,500\,272\,\alpha^{12} + 23\,112\,095\,925\,472\,389\,\alpha^{13} + 4\,737\,102\,973\,509\,780\,\alpha^{14} +\\
&\quad 767\,930\,664\,461\,310\,\alpha^{15} + 96\,195\,146\,877\,576\,\alpha^{16} + 8\,977\,485\,504\,456\,\alpha^{17} +\\
&\quad 587\,451\,930\,408\,\alpha^{18} + 24\,041\,253\,600\,\alpha^{19} + 462\,944\,160\,\alpha^{20}\big)\ \mathrm{Seq}[4+\alpha]
\end{aligned}
$$