Multi-headed Lattice Green Function (N = 4, M = 3)

ODE for $\tilde{R}_{3,4}(z)$ in Theorem 4.2

Out[•]=

$$
\begin{aligned}
&\left(-756-658\,107\,072\,z+749\,920\,296\,960\,z^2-111\,850\,497\,389\,887\,488\,z^3+\right.\\
&\quad 166\,498\,086\,762\,886\,201\,344\,z^4+781\,156\,297\,810\,381\,520\,240\,640\,z^5-\\
&\quad 1\,637\,147\,560\,168\,901\,326\,135\,099\,392\,z^6+732\,231\,540\,023\,730\,620\,969\,986\,818\,048\,z^7+\\
&\quad 149\,697\,886\,463\,404\,317\,932\,617\,973\,366\,784\,z^8-96\,719\,208\,505\,536\,419\,841\,142\,621\,892\,247\,552\,z^9+\\
&\quad 17\,986\,272\,310\,903\,846\,816\,671\,667\,502\,362\,656\,768\,z^{10}-\\
&\quad 1\,365\,121\,772\,758\,889\,406\,361\,975\,817\,513\,893\,625\,856\,z^{11}-\\
&\quad 266\,982\,380\,934\,205\,139\,034\,767\,194\,888\,213\,610\,102\,784\,z^{12}+\\
&\quad 3\,451\,537\,920\,763\,342\,815\,087\,344\,036\,232\,525\,206\,519\,808\,z^{13}+\\
&\quad 57\,677\,198\,298\,608\,369\,079\,887\,772\,175\,160\,628\,008\,189\,952\,z^{14}-\\
&\quad 655\,787\,571\,926\,373\,008\,384\,765\,243\,492\,546\,204\,588\,310\,528\,z^{15}+\\
&\quad \left.241\,642\,117\,251\,606\,275\,763\,798\,810\,128\,911\,651\,647\,258\,624\,z^{16}\right)\,\theta_z^8+\\
&\left(1512+1\,985\,050\,368\,z+6\,350\,035\,230\,720\,z^2+557\,203\,438\,524\,432\,384\,z^3+\right.\\
&\quad 587\,179\,135\,837\,113\,679\,872\,z^4-6\,373\,155\,470\,045\,165\,823\,983\,616\,z^5+\\
&\quad 1\,292\,611\,484\,404\,060\,407\,000\,465\,408\,z^6+5\,768\,579\,650\,367\,308\,639\,843\,349\,692\,416\,z^7-\\
&\quad 1\,382\,427\,013\,757\,614\,047\,365\,601\,869\,955\,072\,z^8-358\,992\,894\,157\,107\,285\,438\,601\,960\,070\,578\,176\,z^9+\\
&\quad 53\,119\,839\,081\,791\,910\,699\,130\,795\,605\,268\,365\,312\,z^{10}-\\
&\quad 20\,033\,688\,987\,472\,672\,410\,340\,587\,286\,921\,796\,911\,104\,z^{11}-\\
&\quad 1\,070\,184\,586\,795\,418\,191\,307\,064\,402\,944\,853\,050\,130\,432\,z^{12}+\\
&\quad 47\,857\,531\,084\,168\,298\,968\,105\,213\,313\,896\,536\,229\,281\,792\,z^{13}+\\
&\quad 270\,884\,195\,420\,762\,288\,774\,538\,561\,958\,062\,360\,455\,282\,688\,z^{14}-\\
&\quad 5\,786\,930\,575\,834\,638\,459\,669\,201\,820\,980\,856\,382\,306\,648\,064\,z^{15}+\\
&\quad \left.1\,933\,136\,938\,012\,850\,206\,110\,390\,481\,031\,293\,213\,178\,068\,992\,z^{16}\right)\,\theta_z^7+\\
&\left(-1113-1\,950\,259\,584\,z-6\,216\,682\,061\,824\,z^2-988\,841\,093\,180\,162\,048\,z^3-\right.\\
&\quad 1\,233\,270\,686\,793\,691\,299\,840\,z^4+9\,715\,936\,946\,399\,911\,434\,256\,384\,z^5+\\
&\quad 7\,752\,074\,094\,169\,934\,619\,780\,055\,040\,z^6+3\,292\,303\,250\,603\,115\,641\,274\,504\,314\,880\,z^7-\\
&\quad 6\,432\,398\,522\,881\,878\,319\,620\,478\,709\,268\,480\,z^8-1\,442\,519\,265\,201\,053\,822\,094\,460\,496\,124\,051\,456\,z^9+\\
&\quad 81\,315\,393\,847\,369\,615\,391\,288\,576\,991\,201\,067\,008\,z^{10}-\\
&\quad 46\,854\,607\,085\,100\,541\,227\,643\,541\,228\,521\,577\,775\,104\,z^{11}-\\
&\quad 484\,722\,323\,648\,843\,742\,960\,229\,713\,378\,845\,655\,040\,000\,z^{12}+\\
&\quad 230\,657\,489\,060\,094\,958\,856\,963\,994\,975\,994\,903\,435\,149\,312\,z^{13}+\\
&\quad 720\,571\,084\,999\,990\,630\,257\,768\,886\,154\,063\,035\,548\,827\,648\,z^{14}-\\
&\quad 22\,027\,520\,447\,398\,165\,760\,511\,055\,419\,885\,169\,351\,394\,852\,864\,z^{15}+\\
&\quad \left.6\,663\,616\,997\,264\,781\,396\,236\,424\,132\,096\,584\,504\,800\,444\,416\,z^{16}\right)\,\theta_z^6+\\
&\left(357+789\,151\,104\,z+10\,558\,964\,416\,512\,z^2+679\,933\,490\,467\,176\,448\,z^3+287\,227\,915\,264\,289\,931\,264\,z^4-\right.\\
&\quad 14\,427\,253\,172\,957\,536\,013\,254\,656\,z^5-3\,092\,284\,696\,452\,480\,308\,007\,141\,376\,z^6-\\
&\quad 905\,311\,608\,923\,360\,926\,047\,701\,827\,584\,z^7-14\,621\,774\,397\,415\,013\,636\,807\,083\,954\,274\,304\,z^8-\\
&\quad 1\,520\,148\,020\,883\,568\,461\,909\,863\,756\,948\,570\,112\,z^9+\\
&\quad 386\,791\,883\,303\,174\,384\,286\,527\,316\,852\,952\,006\,656\,z^{10}-\\
&\quad 55\,223\,977\,247\,937\,670\,737\,556\,473\,181\,100\,776\,095\,744\,z^{11}+\\
&\quad 3\,961\,198\,864\,716\,838\,655\,960\,160\,400\,530\,693\,479\,727\,104\,z^{12}+\\
&\quad 595\,909\,152\,288\,030\,158\,390\,074\,172\,187\,987\,674\,363\,068\,416\,z^{13}+\\
&\quad 1\,344\,613\,983\,895\,642\,776\,006\,711\,946\,247\,382\,448\,056\,827\,904\,z^{14}-\\
&\quad 46\,737\,381\,677\,309\,460\,398\,827\,589\,903\,959\,671\,231\,607\,734\,272\,z^{15}+\\
&\quad \left.12\,917\,784\,851\,408\,785\,491\,873\,078\,058\,141\,402\,044\,309\,700\,608\,z^{16}\right)\,\theta_z^5+\\
&\left(-42-103\,505\,472\,z-4\,717\,152\,813\,056\,z^2-229\,951\,271\,138\,492\,416\,z^3-\right.
\end{aligned}
$$

$$
\begin{aligned}
&33\,452\,654\,058\,350\,313\,472\,z^4 + 3\,519\,629\,264\,891\,117\,955\,973\,120\,z^5 + \\
&8\,712\,355\,168\,877\,862\,347\,467\,653\,120\,z^6 - 3\,183\,462\,774\,294\,546\,535\,677\,280\,911\,360\,z^7 - \\
&12\,662\,024\,101\,532\,041\,005\,416\,571\,287\,371\,776\,z^8 - 91\,457\,574\,708\,638\,075\,983\,201\,720\,533\,516\,288\,z^9 + \\
&586\,373\,716\,393\,067\,719\,463\,798\,499\,745\,499\,971\,584\,z^{10} - \\
&45\,402\,712\,266\,053\,628\,419\,392\,613\,379\,787\,913\,691\,136\,z^{11} + \\
&9\,548\,218\,855\,973\,838\,530\,825\,534\,106\,648\,111\,229\,173\,760\,z^{12} + \\
&910\,171\,319\,762\,953\,713\,098\,938\,394\,074\,694\,947\,157\,573\,632\,z^{13} + \\
&1\,695\,248\,459\,973\,650\,411\,462\,298\,355\,247\,964\,543\,229\,362\,176\,z^{14} - \\
&60\,316\,818\,440\,945\,087\,853\,828\,024\,483\,516\,860\,568\,289\,935\,360\,z^{15} + \\
&15\,391\,679\,930\,285\,039\,325\,517\,386\,362\,534\,096\,505\,705\,332\,736\,z^{16}\big)\,\theta_z^4 + \\
\big(&212\,352\,z + 41\,049\,243\,648\,z^2 - 8\,757\,517\,736\,738\,816\,z^3 + 20\,173\,834\,021\,513\,461\,760\,z^4 - \\
&285\,697\,925\,187\,496\,921\,006\,080\,z^5 + 5\,192\,831\,041\,959\,280\,753\,355\,259\,904\,z^6 - \\
&1\,854\,167\,396\,972\,337\,514\,541\,117\,079\,552\,z^7 - 7\,611\,731\,718\,211\,226\,366\,541\,506\,826\,731\,520\,z^8 + \\
&1\,132\,092\,616\,093\,392\,427\,901\,870\,092\,654\,215\,168\,z^9 + \\
&573\,360\,955\,845\,607\,449\,871\,633\,552\,338\,403\,196\,928\,z^{10} - \\
&31\,196\,974\,018\,934\,147\,496\,719\,981\,299\,967\,906\,021\,376\,z^{11} + \\
&10\,343\,484\,480\,536\,631\,324\,792\,615\,211\,856\,556\,940\,328\,960\,z^{12} + \\
&850\,871\,471\,160\,179\,197\,799\,997\,784\,539\,641\,065\,269\,886\,976\,z^{13} + \\
&1\,366\,110\,491\,586\,634\,438\,250\,598\,685\,972\,842\,495\,483\,052\,032\,z^{14} - \\
&48\,462\,627\,453\,914\,171\,613\,580\,503\,834\,409\,896\,539\,840\,315\,392\,z^{15} + \\
&11\,533\,376\,887\,988\,124\,536\,976\,314\,041\,777\,845\,706\,747\,281\,408\,z^{16}\big)\,\theta_z^3 + \\
\big(&102\,144\,z - 45\,890\,052\,096\,z^2 - 4\,372\,668\,181\,905\,408\,z^3 + 25\,373\,328\,015\,678\,767\,104\,z^4 + \\
&35\,860\,603\,273\,980\,739\,059\,712\,z^5 + 2\,609\,042\,215\,039\,715\,330\,989\,490\,176\,z^6 - \\
&1\,135\,805\,724\,897\,588\,664\,940\,548\,325\,376\,z^7 - 2\,255\,387\,710\,140\,891\,706\,830\,918\,298\,632\,192\,z^8 + \\
&1\,208\,690\,199\,949\,684\,174\,443\,411\,490\,448\,867\,328\,z^9 + \\
&334\,887\,474\,030\,943\,944\,488\,261\,929\,148\,495\,167\,488\,z^{10} - \\
&17\,877\,519\,858\,996\,120\,053\,115\,971\,187\,944\,045\,150\,208\,z^{11} + \\
&6\,128\,654\,166\,961\,763\,785\,820\,170\,570\,933\,495\,910\,105\,088\,z^{12} + \\
&476\,288\,752\,718\,822\,257\,140\,265\,748\,951\,023\,617\,535\,115\,264\,z^{13} + \\
&668\,324\,955\,523\,996\,949\,091\,967\,282\,097\,867\,454\,465\,179\,648\,z^{14} - \\
&23\,673\,396\,984\,425\,987\,991\,182\,604\,909\,788\,067\,572\,173\,766\,656\,z^{15} + \\
&5\,303\,121\,535\,030\,477\,312\,378\,786\,039\,652\,035\,049\,432\,285\,184\,z^{16}\big)\,\theta_z^2 + \\
\big(&18\,816\,z - 15\,679\,168\,512\,z^2 - 1\,105\,852\,812\,492\,800\,z^3 + 9\,253\,977\,260\,438\,847\,488\,z^4 + \\
&36\,631\,485\,914\,913\,630\,584\,832\,z^5 + 726\,314\,268\,655\,758\,437\,624\,315\,904\,z^6 - \\
&358\,748\,918\,263\,218\,897\,800\,795\,258\,880\,z^7 - 286\,918\,040\,829\,362\,957\,086\,349\,485\,670\,400\,z^8 + \\
&503\,093\,135\,988\,065\,408\,878\,446\,537\,502\,359\,552\,z^9 + \\
&108\,054\,624\,128\,516\,395\,031\,347\,156\,140\,800\,606\,208\,z^{10} - \\
&6\,462\,199\,176\,714\,597\,967\,385\,137\,880\,595\,550\,961\,664\,z^{11} + \\
&1\,918\,694\,308\,581\,208\,774\,434\,293\,647\,882\,766\,231\,011\,328\,z^{12} + \\
&145\,968\,214\,956\,821\,518\,855\,061\,140\,764\,657\,290\,013\,835\,264\,z^{13} + \\
&180\,192\,916\,196\,793\,417\,299\,520\,478\,544\,551\,103\,988\,498\,432\,z^{14} - \\
&6\,427\,990\,896\,954\,765\,589\,117\,223\,123\,312\,617\,887\,797\,084\,160\,z^{15} + \\
&1\,366\,788\,225\,704\,397\,997\,288\,987\,019\,791\,656\,529\,629\,806\,592\,z^{16}\big)\,\theta_z + \\
\big(&1344\,z - 1\,639\,464\,960\,z^2 - 114\,267\,116\,273\,664\,z^3 + 1\,211\,269\,289\,902\,866\,432\,z^4 + \\
&5\,825\,052\,469\,481\,755\,901\,952\,z^5 + 84\,152\,329\,059\,287\,491\,751\,706\,624\,z^6 - \\
&48\,938\,139\,253\,071\,191\,076\,992\,188\,416\,z^7 - 3\,045\,898\,181\,345\,513\,899\,617\,530\,413\,056\,z^8 + \\
&78\,022\,182\,208\,697\,643\,235\,066\,215\,175\,028\,736\,z^9 + \\
&14\,678\,268\,634\,598\,917\,861\,557\,009\,824\,329\,236\,480\,z^{10} - \\
&991\,390\,991\,530\,383\,611\,754\,057\,315\,362\,342\,436\,864\,z^{11} + \\
&247\,958\,505\,832\,498\,167\,951\,336\,010\,415\,935\,397\,560\,320\,z^{12} + \\
&18\,747\,996\,529\,475\,474\,000\,600\,656\,049\,610\,020\,358\,193\,152\,z^{13} +
\end{aligned}
$$

$$20\,499\,222\,726\,707\,352\,515\,629\,191\,626\,716\,497\,397\,678\,080\,z^{14} - 742\,685\,376\,897\,284\,273\,453\,811\,376\,847\,779\,469\,564\,313\,600\,z^{15} + 151\,026\,323\,282\,253\,922\,352\,374\,256\,330\,569\,782\,279\,536\,640\,z^{16}\Big)$$